CONCENTRATION IN A SUSPENSION DISCHARGING FROM A MIXED VESSEL

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Theoretical model has been proposed for the calculation of concentration of a suspension discharging at the top from a mixed vessel. It has been assumed that no separation of solids occurs during sampling near the level and that the samples provide local values of concentration obeying the diffusional model. The model has been tested on experiments involving a cylindrical vessel with baffles mixed by a propeller impeller.

The design of a continuous reactor for reactions involving solids and a liquid requires the knowledge of the ratio of the outlet concentration of the suspension and the average concentration within the reactor. Procedures have been described in the literature for the calculation of this concentration ratio under idealized conditions of sampling – the so-called isokinetic sampling. In practice, of course, the suspension discharges at the top of the mixed vessel, most often *via* an overflow weir.

This paper is an attempt to calculate the concentration ratio of a suspension sampled at the top of the vessel. The description starts from the idea that in the top part of the mixed vessel there is a region of relatively quiescent liquid free of abrupt changes of the velocity and the direction of the liquid flow. Further it is assumed that no separation of the solids from liquid occurs in this part and that the composition of the samples corresponds to local concentration. For the calculation of the local concentration in the suspension a two-dimensional model is proposed.

THEORETICAL

The model utilizes the idea that the solid particles are kept suspended by the diffusional mechanism, characterized by the diffusion coefficient. A solid particle is assumed to be under the effect of gravity in the axial direction and of the centrifugal force in the radial direction induced by the rotational motion of the suspension.

Mathematically this is formulated by the following equation

$$(-v_h + v_r)\, c = \mathscr{D}(\partial c/\partial h) + \partial c/\partial r)\,, \qquad (1)$$

the boundary condition

$$\int_0^H \int_0^{D_N/2} rc(r, h)\, dr\, dh = 1/8 D_N^2 H \bar{c} \tag{2}$$

and the balance of forces

$$1/6 \pi d^3 (\varrho_p - \varrho_c)\, g = 1/4 \pi C_d d^2 \varrho_c v_h^2 \,, \tag{3}$$

$$1/6 \pi d^3 (\varrho_p - \varrho_c)\, r\omega^2 = 1/4 \pi C_d d^2 \varrho_c v_r^2 \,. \tag{4}$$

The axial velocity component, v_h, with respect to the chosen frame of reference is taken positively. The distribution of concentration of the suspension within the mixed vessel is given by the set of Eqs (*1*)–(*4*). A solution can be obtained by separation to two ordinary differential equations and it is sought in the form

$$c(h, r) = \text{const.}\ \varphi(h)\,.\,\psi(r)\,. \tag{5}$$

The axial profile of the suspension concentration $\varphi(h)$ is given by the solution of the following equation

$$\mathscr{D}\, dc/dh = -v_h c\,. \tag{6}$$

The general solution after introducing dimensionless variables as

$$Z = h/H\,, \quad P = H v_h/\mathscr{D} \tag{7), (8}$$

takes the form

$$\varphi(Z) = \text{const.}_1\,.\,\exp\left(-P\,.\,Z\right). \tag{9}$$

The radial profile $\psi(r)$ is given by the solution of the equation

$$\mathscr{D}\, dc/dr = v_r c\,. \tag{10}$$

Solution of Eq. (*10*) necessitates the knowledge of the radial velocity component to be found from Eq. (*3*) and (*4*)

$$v_r = v_h \omega r^{0.5}\, g^{-0.5}\,. \tag{11}$$

After introducing dimensionless parameters

$$Q = (D_N^{3/2} v_h \omega)/(2^{1.5}\, \mathscr{D}\, g^{0.5}),\ X = 2r/D_N \tag{12}$$

the general solution of Eq. (*10*) takes the form

$$\psi(X) = \text{const.}_2 \,.\, \exp\left(2/3QX^{3/2}\right). \tag{13}$$

Combining Eqs (*5*), (*9*) and (*13*) one obtains

$$c(Z, X) = \text{const.} \exp\left(-P\,Z\right) . \exp\left(2/3 \,.\, QX^{3/2}\right). \tag{14}$$

The constant is found by substituting for concentration from Eq. (*14*) into the boundary condition (*2*)

$$\text{const.} \int_0^1 \int_0^1 X \exp\left(-PZ\right) . \exp\left(2/3 \,.\, QX^{3/2}\right) \mathrm{d}Z \,\mathrm{d}X = 1/2\bar{c}\,. \tag{15}$$

The Simpson rule was used to evaluate the second integral and to find the constant from Eq. (*15*)

$$\text{const.} = \left[\bar{c} \,.\, P/(1 - \exp\left(-P\right))\right] . 3/\left[2 \exp\ \left(\sqrt{2}Q/6\right) + \exp\left(2Q/3\right)\right]. \tag{16}$$

Combining Eqs (*14*) and (*16*) one obtains finally the concentration distribution of the solids as

$$c(Z, X) = \left[\bar{c}P \exp\left(-PZ\right)/(1 - \exp\left(-P\right))\right] . \left[3 \exp\left(2QX^{3/2}/3\right)\right]/\left[2 \exp\left(\sqrt{2}Q/6\right) + \right.$$
$$\left. + \exp\left(2Q/3\right)\right]. \tag{17}$$

The first term in Eq. (*17*) determines the radial profile of concentration, the second term represents a correction on the effect of the centrifugal force induced by the rotational motion of the suspension.

For sampling at the wall of the vessel ($X = 1$) Eq. (*17*) changes to

$$c(Z, 1)/\bar{c} = \left[P \,.\, \exp\left(-PZ\right)/(1 - \exp\left(-P\right))\right] . \left[3/\left[1 + 2 \exp\left(\sqrt{2}/6 - 2/3\right) Q\right]\right]. \tag{18}$$

To be able to compare experimental values of $c/\bar{c}$ with Eq. (*18*), the right hand side of the last equation had to be rearranged in order to contain directly measurable parameters. The diffusion coefficient was assumed to be a linear function of the frequency of revolution of the impeller and a power-law type function of particle diameter

$$\mathscr{D} \sim nD_{\mathrm{M}}^2(d/D_{\mathrm{M}})^{k_3}. \tag{19}$$

Substituting for the diffusion coefficient from Eq. (*19*) into Eq. (*8*) the following expression for the Peclet number was obtained

$$P = (k_1 v_h H / n D_M^2)(D_M/d)^{k_3} . \qquad (20)$$

In order to express the parameter Q, the angular velocity was assumed to be a linear function of the frequency of revolution of the impeller

$$\omega \sim n \qquad (21)$$

and by combining Eqs (*8*), (*12*), (*20*) and (*21*) the following relation was obtained

$$Q = k_2 v_h \,.\, (D_M/d)^{k_3} \, g^{-0.5} D_M^{-0.5} . \qquad (22)$$

EXPERIMENTAL

The experimental set-up consisted of a cylindrical vessel 298 mm in diameter, 298 mm high, mixed by a standard propeller-type impeller 91·5 mm in diameter (lead to diameter ratio 1 : 1) located 100 mm above the bottom. The vessel was equipped with four standard radial baffles 29·8 mm wide. The whole vessel was filled by liquid in order to prevent formation of the central vortex. The suspension discharged through a 26 mm in diameter tube passing through the wall between two baffles. The dynamic method described elsewhere[1] was used to determine the ratio of the outlet concentration of the suspension to the mean concentration in the vessel. The measurements were carried out for two different distances of the discharge opening from the bottom: $Z = 0{\cdot}92$ and $Z = 0{\cdot}72$, for the range of frequency of revolution between 10 and 43 s^{-1} and for the flow rate of the suspension ranging between $0{\cdot}3 \,.\, 10^{-4}$ and $1 \,.\, 10^{-4}$ $m^3 s^{-1}$ (mean residence time of the suspension in the vessel ranged between $2 \,.\, 10^2$ and $7 \,.\, 10^2$ s). The mean concentration of the solid particles within the batch was always less than 5% by volume. Physical properties of the system are summarized in Table I.

TABLE I

Physical Properties of Investigated Systems

System	Solids	$d_e \,.\, 10^3$, m	$\varrho_p \,.\, 10^{-3}$, kg m^{-3}	Liquid	$\nu \,.\, 10^6$, $m^2 s^{-1}$	$v_0 \,.\, 10^2$, ms^{-1}
1	Perspex glass	0·55	1·166	water	0·89	1·9
2	glass	0·18	2·655	water	0·89	1·98
3	sugar	0·605	1·586	kerosene	1·56	5·7
4	glass	0·45	2·655	water	0·89	8
5	glass	0·9	2·655	water	0·89	15·2

The densities of the solids and the liquid were measured by pycnometry; viscosities in the Hoeppler viscosimeter. Terminal velocities of the solid particles were measured in a quiescent liquid. The diameter taken to characterize the solid particles is the equivalent diameter computed from the experimental terminal velccities.

RESULTS AND DISCUSSION

The experimental values $c/\bar{c}$ did not display in the examined range any dependence on the flow rate of the suspension and, accordingly, the data for all flow rates were evaluated simultaneously. The particle velocities in the axial direction, v_h, used were those of the terminal velocities v_0, measured in the quiecent liquid. The constants k_1, k_2, k_3 were evaluated from experimental data numerically on a computer using Marquardt's optimization technique. The optimum values found were as follows: $k_1 = 4{\cdot}82$, $k_2 = 0{\cdot}98$, $k_3 = 0{\cdot}37$ with the mean square deviation of the experimental and computed data being 0·0515. For the sampling point close to the liquid level the accuracy of the approximation is greater than corresponds to the above indicated deviation. A comparison of the computed and experimental data for the sampling point near the liquid level $(Z = 0{\cdot}92)$ is shown in Fig. 1. For samplings deeper below the liquid level $(Z = 0{\cdot}72)$ the deviations of the computed and experimental data are greater. This case probably violates the basic assumption of the model, namely, the assumption of local concentration sampling. Owing to the more intensive streaming in this region, the liquid in the discharge opening is forced to change suddenly its direction and velocity of the flow and by the action of the inertia forces separation of the solids from the liquid takes place. The diffusion coefficient of the proposed model has been compared with the turbulent diffusivity as it is defined in the theory of turbulence. The comparison can be only a qualitative one as we have only power input characteristics averaged over the whole column of the batch at our disposal. Thus we cannot expect local isotropy of the turbulent motion of the suspension.

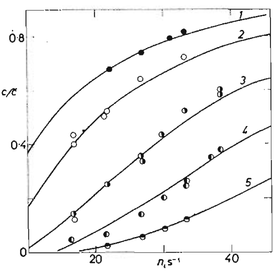

FIG. 1

Experimental and Computed Data, $Z = 0{\cdot}92$
System: ● *1*, ○ *2*, ◑ *3*, ◐ *4*, ◓ *5*.

Assuming the existence of an average vortex of size l_e, the turbulent diffusivity is defined by

$$\mathscr{D}_t = \bar{v} l_M \,, \tag{23}$$

where the Prandtl's mixing length, l_M, is usually expressed from analogy with the turbulent flow of gases by

$$l_M = 1{\cdot}5 l_e \,. \tag{24}$$

Kuboi, Komasawa and Otake[2] measured the mean fluctuation velocity of particles suspended by a turbine impeller and proposed the following correlation for its calculation

$$\overline{v^2} = 2(\varepsilon d)^{2/3} \,. \tag{25}$$

The rate of energy dissipation may be expressed from the power input on the impeller

$$\varepsilon = 0{\cdot}3 n^3 D_M^5 / (D_N^2 H) \,. \tag{26}$$

Combining Eqs (23)–(26) and substituting for geometrical parameters of the vessel, the following relationship for the turbulent diffusivity is obtained

$$\mathscr{D}_t = 8{\cdot}9 \,.\, 10^{-2} \, l_e n d^{1/3} \,. \tag{27}$$

The last relationship is in agreement with the anticipated linear dependence of the diffusion coefficient on the frequency of revolution of the impeller. Equally the exponent over the particle diameter is in good agreement with the found value $k_3 = 0{\cdot}37$. The parameter k_3 thus may be regarded to be independent of the geometry of the system.

The literature data indicate[2] that solid particles falling in a turbulent liquid experience an increased resistance owing to the turbulent motion of the liquid. It is apparent that the increase of the resistance coefficient affects the terminal velocity of the particles and also the turbulent diffusivity of the particles. Peňáz, Rod and Řeháková[4] evaluated the ratio of the terminal velocity of a particle in a vessel mixed by a paddle impeller to that in a quiescent liquid as a parameter of the model of the spatial distribution of concentration of solids. The value of this parameter decreased with increasing frequency of revolution of the impeller. Schwartzberg and Treybal[3] examined experimentally the ratio of these velocities in a vessel mixed by a turbine impeller and published a value ranging between 0·3 and 0·5. These data, however, possess largely a qualitative character and do not permit a closer analysis of this phenomenon. For this reason in the evaluation of our experimental data we substituted for the terminal velocity v_h in the Peclet number the terminal velocity of the particle in a quiescent liquid, v_0 and assumed that the effect of changing resistance coefficient shall be accounted for in the mean value of the evaluated constant k_1. With the aid of the mean value of the velocity ratio found by Schwartzberg and Treybal[3], $v_h/v_0 = 0{\cdot}4$, and after substitution for the constant k_1 the value found by evaluation, $k_1 = 4{\cdot}82$, a combination of Eqs (8), (19), (20) and (27) leads to the following value of the mean size of the vortex $l_e = 0{\cdot}016$ m. Alternatively, referred to the impeller diameter

$$l_e = 0{\cdot}16 D_M\,. \tag{28}$$

A comparison of this formula with the equation

$$l_e = 0{\cdot}08 D_M \tag{29}$$

which has been used to express the size of the vortices in close proximity of the turbine impeller[3] indicates that the mean effective size of the vortex characterizing the intensity of turbulence in a mixed vessel is twice the scale of the vortices in the immediate vicinity of the impeller.

The constant k_3 expresses proportionality between angular velocity of the suspension and the frequency of revolution of the impeller. Substituting for ω the angular velocity of the blade of the impeller and for the diffusion coefficient from Eqs (8), (19) and (20) using the constant $k_1 = 4{\cdot}82$ and the ratio $v_h/v_0 = 0{\cdot}4$, one obtains for k_2 the value 84. A comparison with k_2 following from experimental data, $k_2 = 0{\cdot}98$, indicates that the mean effective angular velocity of the suspension is about 1% of the angular velocity of the impeller. From the agreement of the experimental data with those following from the model it can be inferred that the assumption of sampling local concentration of the suspension at the top of a mixed vessel is valid and may be used in the calculations. The calculation requires knowledge of two parameters of the diffusional model, which are independent of the geometry of the system. The parameter k_1 expresses the dependence of the diffusion coefficient on the frequency of revolution of the impeller while the parameter k_2 the dependence of angular velocity of the suspension ω on the frequency of revolution of the impeller.

LIST OF SYMBOLS

c	concentration of solids, kg m^{-3}
$\bar{c}$	mean concentration of solids in vessel, kg m^{-3}
C_d	drag coefficient
$\mathcal{D}$	diffusion coefficient, m^2 s^{-1}
$\mathcal{D}_t$	turbulent diffusivity, m^2 s^{-1}
D_M	impeller diameter, m
D_N	vessel diameter, m

d	particle diameter
d_e	equivalent particle diameter, m
g	acceleration due to gravity, ms^{-2}
h	axial coordinate, m
H	height of vessel, m
k_1, k_2, k_3	constants
l_M	mixing length, m
l_e	scale of mean vortex, m
n	frequency of revolution of impeller, s^{-1}
P	Peclet number
Q	dimensionless criterion (Eq. (*12*))
r	radial coordinate, m
v_r	radial velocity component of a solid particle, $m s^{-1}$
v_h	axial velocity component of solid particle, $m s^{-1}$
v_0	terminal velocity in a quiescent liquid, $m s^{-1}$
$\bar{v}$	mean velocity of vertex, $m s^{-1}$
X	dimensionless radial coordinate
Z	dimensionless axial coordinate
ε	rate of energy dissipation per unit mass, $m^2 s^{-3}$
ν	kinematic viscosity, $m^2 s^{-1}$
ϱ_p	density of solids, $kg m^{-3}$
ϱ_c	density of liquid, $kg m^{-3}$
φ	axial concentration distribution function, kgm^{-3}
ψ	radial concentration distribution function, kgm^{-3}
ω	angular velocity, s^{-1}

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